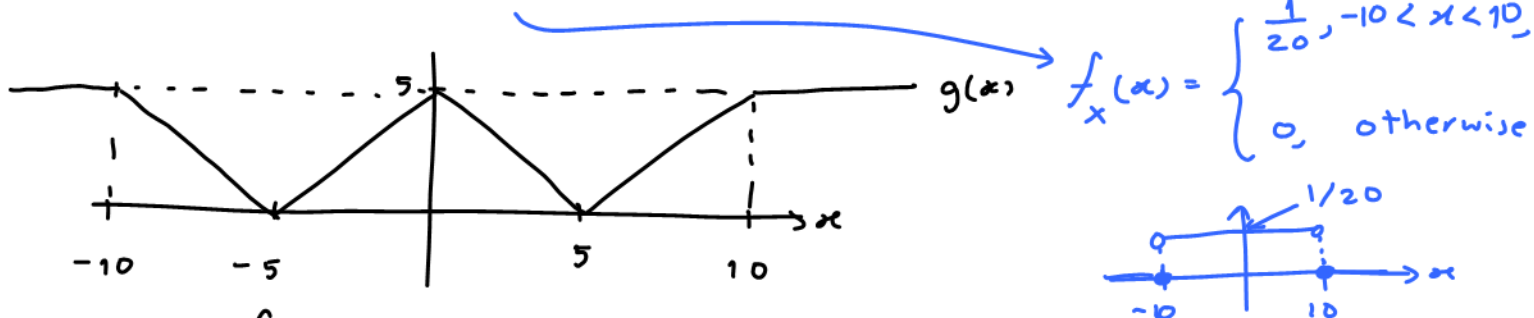
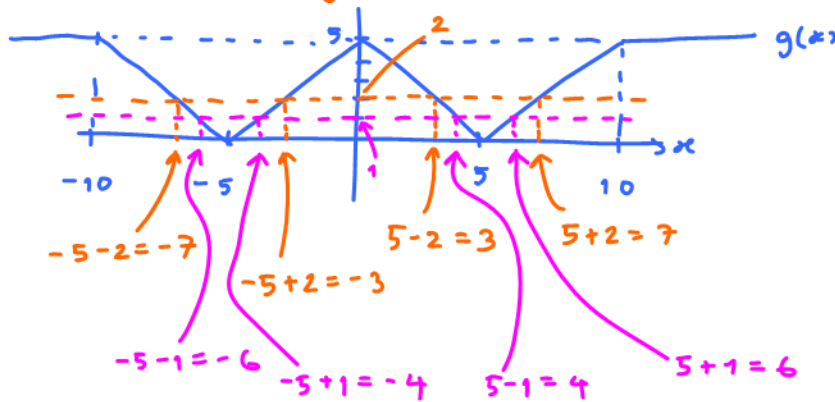


Quiz # 6. $X \sim U(-10, 10)$. Let $Y = g(X)$.



(a) Find $f_Y(2)$.

① Find root(s) of $g(x) = 2$. $\Rightarrow x = \pm 3, \pm 7$



② Find slopes: at $x = -3, 7$, $g'(x) = 1$.

at $x = 3, -7$, $g'(x) = -1$.

$$\begin{aligned} \textcircled{3} f_Y(2) &= \frac{f_X(-7)}{|g'(-7)|} + \frac{f_X(-3)}{|g'(-3)|} + \frac{f_X(3)}{|g'(3)|} + \frac{f_X(7)}{|g'(7)|} = \frac{1/20}{|-1|} + \frac{1/20}{|1|} + \frac{1/20}{|-1|} + \frac{1/20}{|1|} \\ &= \frac{1}{20} \times 4 = \frac{1}{5}. \end{aligned}$$

(b) Find $f_Y(1)$.

Find root(s) of $g(x) = 1 \Rightarrow x = \pm 4, \pm 6$

$$f_Y(1) = \frac{f_X(-6)}{|g'(-6)|} + \frac{f_X(-4)}{|g'(-4)|} + \frac{f_X(4)}{|g'(4)|} + \frac{f_X(6)}{|g'(6)|}$$

$$= \frac{1/20}{|-1|} + \frac{1/20}{|1|} + \frac{1/20}{|-1|} + \frac{1/20}{|1|} = \frac{1}{20} \times 4 = \frac{1}{5}$$

(c) Repeat (a) and (b) but use $X \sim E(1)$. $f_X(x) = \begin{cases} e^{-x} & x > 0, \\ 0 & \text{otherwise} \end{cases}$

$$\text{(a)} f_Y(2) = \frac{f_X(-7)}{|g'(-7)|} + \frac{f_X(-3)}{|g'(-3)|} + \frac{f_X(3)}{|g'(3)|} + \frac{f_X(7)}{|g'(7)|} = \frac{e^{-3}}{1} + \frac{e^{-7}}{1} \approx 0.0507$$

$$\text{(b)} f_Y(1) = \frac{f_X(-6)}{|g'(-6)|} + \frac{f_X(-4)}{|g'(-4)|} + \frac{f_X(4)}{|g'(4)|} + \frac{f_X(6)}{|g'(6)|} = \frac{e^{-4}}{1} + \frac{e^{-6}}{1} \approx 0.0208$$